

Logarithmic functions

Graphs of Logarithmic Functions (pg. 604-608)

Consider the exponential function $f(x) = 2^x$. Like all exponential functions, f is one-to-one. Can you find a formula for the inverse of the function? The inverse of the function is written as $f^{-1}(x)$

1. Replace $f(x)$ with y . $y = 2^x$
2. Interchange x and y . $x = 2^y$
3. Solve for y $y =$ the exponent to which we raise 2 to get x .
4. Replace y with $f^{-1}(x)$ $f^{-1}(x) =$ the exponent to which we raise 2 to get x .

We define a new symbol to replace the words: “The exponent to which we raise 2 to get x ”
 $\log_2 x$ read “the logarithm, base 2, of x ” or “log, base 2, of x ”, means “the exponent to which we raise 2 to get x ”

If $b > 0$ and $b \neq 1$, then $y = \log_b x$ means $x = b^y$ for every $x > 0$ and every real number y .

$y = \log_b x$ (read as “ y is the logarithm to the base b of x ”)

The domain of the logarithm function is $x > 0$.

Examples:

Simplify:

$$\log_2 32 \qquad \log_2 1 \qquad \log_2 \frac{1}{8}$$

The Meaning of $y = \log_a x$

For $x > 0$ and a a positive constant other than 1, $y = \log_a x$ is the exponent to which a must be raised in order to get x . Thus, $\log_a x = m$ means $a^m = x$.

Example:

Simplify:

$$7^{\log_7 85}$$

Graph $y = f(x) = \log_5 x$

Equivalent Equations: $\log_a x = m$ means $a^m = x$

Examples:

Rewrite each as an equivalent exponential equation

$$y = \log_3 5 \qquad -2 = \log_a 7 \qquad a = \log_b d$$

Rewrite each as an equivalent logarithmic equation

$$8 = 2^x \qquad y^{-1} = 4 \qquad a^b = c$$

Solve

$$\log_2 x = -3 \qquad \log_x 16 = 2$$

The Principle of Exponential Equality

For any real number b , where $b \neq -1, 0$, or 1 , $b^{x_1} = b^{x_2}$ is equivalent to $x_1 = x_2$
(Powers of the same base are equal if and only if the exponents are equal.)

Examples:

Solve:

$$\log_{10} 1000 = x \qquad \log_4 1 = t$$

Properties of Logarithms

If b is a real number, $b > 0$ and $b \neq 1$ then

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$